

Algebra Review Notes

Name: Key

Solving Systems of Linear Equations

Date: _____ Pd: _____

Solving Systems by Substitution

- Procedure:
1. Solve one of the equations for one of the variables.
 2. Substitute the expression for the variable into the second equation.
 3. Solve for the remaining variable in the second equation.
 4. Substitute this value into the revised first equation and solve for the second variable. The solution is a set of coordinates.
 5. Check the solution in both equations.

Example 1:
$$\begin{cases} x + y = 1 \\ 2x - 3y = 12 \end{cases}$$

Solve the first equation for y .

$$y = -x + 1$$

Substitute $-x + 1$ for y in the second equation.

$$2x - 3(-x + 1) = 12$$

Distribute -3 .

$$2x + 3x - 3 = 12$$

Simplify and solve for x .

$$5x - 3 = 12$$

$$5x = 15$$

$$x = 3$$

Substitute $x = 3$ into the revised first equation and solve.

$$y = -(3) + 1$$

$$y = -2$$

The solution is $(3, -2)$. Check the ordered pair with both equations.

$$3 + (-2) = 1 \quad 2(3) - 3(-2) = 12 \quad \text{The solution makes both equations true.}$$

Example 2:
$$\begin{cases} 2x + 3y = 7 \\ x + 2y = 4 \end{cases}$$

Solve the second equation for x .

$$x = -2y + 4$$

Substitute $-2y + 4$ for x in the first equation.

$$2(-2y + 4) + 3y = 7$$

Distribute 2.

$$-4y + 8 + 3y = 7$$

Simplify and solve for y .

$$-y + 8 = 7$$

$$-y = -1$$

$$y = 1$$

Substitute $y = 1$ into the revised second equation and solve.

$$x = -2(1) + 4$$

$$x = 2$$

The solution is $(2, 1)$. Check the ordered pair with both equations.

$$2(2) + 3(1) = 7 \quad 2 + 2(1) = 4 \quad \text{The solution makes both equations true.}$$

Solve each system of linear equations by substitution. Check and circle your solution.

$$1. \begin{cases} \textcircled{1} x+2y=-5 \\ \textcircled{2} 4x-3x=2 \end{cases}$$

$$\textcircled{2} x=2$$

$$\textcircled{1} 2+2y=-5$$

$$2y=-7$$

$$y=-\frac{7}{2}$$

$$\boxed{(2, -\frac{7}{2})} \checkmark$$

$$2. \begin{cases} \textcircled{1} 3x-2y=4 \\ \textcircled{2} x+3y=5 \end{cases}$$

$$\textcircled{2} x=-3y+5$$

$$\textcircled{1} 3(-3y+5)-2y=4$$

$$-9y+15-2y=4$$

$$-11y=-11$$

$$y=1$$

$$\textcircled{2} x=-3(1)+5$$

$$x=-3+5$$

$$x=2$$

$$\boxed{(2, 1)} \checkmark$$

$$3. \begin{cases} \textcircled{1} 3x+y=-2 \\ \textcircled{2} x+3y=2 \end{cases}$$

$$\textcircled{1} y=-3x-2$$

$$\textcircled{2} x+3(-3x-2)=2$$

$$x+(-9x)-6=2$$

$$-8x=8$$

$$x=-1$$

$$\textcircled{1} y=-3(-1)-2$$

$$y=3-2$$

$$y=1$$

$$\boxed{(-1, 1)} \checkmark$$

$$4. \begin{cases} \textcircled{1} y=3x+1 \\ \textcircled{2} y=4x+5 \end{cases}$$

$$\textcircled{1} 4x+5=3x+1$$

$$x=-4$$

$$\textcircled{1} y=3(-4)+1$$

$$y=-12+1$$

$$y=-11$$

$$\boxed{(-4, -11)} \checkmark$$

$$5. \begin{cases} \textcircled{1} 3x-y=5 \\ \textcircled{2} x=2y+10 \end{cases}$$

$$\textcircled{1} 3(2y+10)-y=5$$

$$6y+30-y=5$$

$$5y=-25$$

$$y=-5$$

$$\textcircled{2} x=2(-5)+10$$

$$x=-10+10$$

$$x=0$$

$$\boxed{(0, -5)} \checkmark$$

$$6. \begin{cases} \textcircled{1} y=-3x+1 \\ \textcircled{2} 2x-y=9 \end{cases}$$

$$\textcircled{2} 2x-(-3x+1)=9$$

$$2x+3x-1=9$$

$$5x=10$$

$$x=2$$

$$\textcircled{1} y=-3(2)+1$$

$$y=-6+1$$

$$y=-5$$

$$\boxed{(2, -5)} \checkmark$$

Solving Systems by Linear Combination

- Procedure:
1. Line up each equation in standard form.
 2. Multiply one or both of the equations by a number that will give you opposite coefficients for one of the variables.
 3. Add the equations together and solve for the remaining variable.
 4. Substitute this value into one of the original equations and solve for the second variable. The solution is a set of coordinates.
 5. Check the solution in both equations.

Example 1:
$$\begin{cases} 3x + 4y = 15 \\ -3x + 2y = 21 \end{cases}$$

The x coefficients are already opposites.
Add the equations together and solve for y .

$$\begin{aligned} 6y &= 36 \\ y &= 6 \end{aligned}$$

Substitute $y = 6$ into the first equation and solve for x .

$$\begin{aligned} 3x + 4(6) &= 15 \\ 3x + 24 &= 15 \\ 3x &= -9 \\ x &= -3 \end{aligned}$$

The solution is $(-3, 6)$. Check the ordered pair with both equations.

$$3(-3) + 4(6) = 15 \quad -3(-3) + 2(6) = 21 \quad \text{The solution makes both equations true.}$$

Example 2:
$$\begin{cases} 4x - 3y = 11 \\ 3x + 2y = -13 \end{cases}$$

Multiply the first equation by 2 and the second equation by 3.

$$\begin{cases} 8x - 6y = 22 \\ 9x + 6y = -39 \end{cases}$$

Add the equations together and solve for x .

$$\begin{aligned} 17x &= -17 \\ x &= -1 \end{aligned}$$

Substitute $x = -1$ into the second equation and solve for y .

$$\begin{aligned} 3(-1) + 2y &= -13 \\ -3 + 2y &= -13 \\ 2y &= -10 \\ y &= -5 \end{aligned}$$

The solution is $(-1, -5)$. Check the ordered pair with both equations.

$$4(-1) - 3(-5) = 11 \quad 3(-1) + 2(-5) = -13 \quad \text{The solution makes both equations true.}$$

Solve each system by linear combination. Check and circle your solution.

$$7. \begin{cases} \textcircled{1} 2x+4y=24 \longrightarrow 2x+4y=24 \\ \textcircled{2} 2x-3y=3 \xrightarrow{x-1} -2x+3y=-3 \end{cases}$$

$$\begin{array}{r} 7y=21 \\ y=3 \end{array}$$

$$\textcircled{1} 2x+12=24$$

$$2x=12$$

$$x=6$$

$$\boxed{(6, 3)} \checkmark$$

$$8. \begin{cases} \textcircled{1} 5x+3y=-12 \xrightarrow{\times 2} 10x+6y=-24 \\ \textcircled{2} -2x+2y=8 \xrightarrow{\times 5} -10x+10y=40 \end{cases}$$

$$\begin{array}{r} 16y=16 \\ y=1 \end{array}$$

$$\textcircled{1} 5x+3=-12$$

$$5x=-15$$

$$x=-3$$

$$\boxed{(-3, 1)} \checkmark$$

$$9. \begin{cases} \textcircled{1} 3x+7y=12 \xrightarrow{\times -2} -6x-14y=-24 \\ \textcircled{2} 5x+14y=20 \longrightarrow 5x+14y=20 \end{cases}$$

$$\begin{array}{r} -x=-4 \\ x=4 \end{array}$$

$$\textcircled{1} 12+7y=12$$

$$7y=0$$

$$y=0$$

$$\boxed{(4, 0)} \checkmark$$

$$10. \begin{cases} \textcircled{1} 2x-3y=-1 \xrightarrow{\times 3} 6x-9y=-3 \\ \textcircled{2} 4x+9y=-17 \longrightarrow 4x+9y=-17 \end{cases}$$

$$\begin{array}{r} 10x=-20 \\ x=-2 \end{array}$$

$$\textcircled{1} -4-3y=-1$$

$$-3y=3$$

$$y=-1$$

$$\boxed{(-2, -1)} \checkmark$$

$$11. \begin{cases} \textcircled{1} 4x-5y=6 \longrightarrow 4x-5y=6 \\ \textcircled{2} 2x+3y=-8 \xrightarrow{\times -2} -4x-6y=16 \end{cases}$$

$$\begin{array}{r} -11y=22 \\ y=-2 \end{array}$$

$$\textcircled{1} 4x+10=6$$

$$4x=-4$$

$$x=-1$$

$$\boxed{(-1, -2)} \checkmark$$

$$12. \begin{cases} \textcircled{1} -7x+8y=32 \xrightarrow{\times 3} -21x+24y=96 \\ \textcircled{2} 5x+6y=24 \xrightarrow{\times -4} -20x-24y=-96 \end{cases}$$

$$\begin{array}{r} -41x=0 \\ x=0 \end{array}$$

$$\textcircled{1} 0+8y=32$$

$$y=4$$

$$\boxed{(0, 4)} \checkmark$$

Solve each system by any method you choose. Check and circle your solution.

$$13. \begin{cases} ① x+y=5 \\ ② x-y=7 \end{cases}$$

$$\underline{2x=12}$$

$$x=6$$

$$① 6+y=5$$

$$y=-1$$

$$\boxed{(6, -1)} \checkmark$$

$$14. \begin{cases} ① -2x+3y=14 \rightarrow -2x+3y=14 \\ ② x-4y=-12 \xrightarrow{\times 2} 2x-8y=-24 \end{cases}$$

$$\underline{-5y=-10}$$

$$y=2$$

$$② x-8=-12$$

$$x=-4$$

$$\boxed{(-4, 2)} \checkmark$$

$$15. \begin{cases} ① 2x-y=3 \xrightarrow{\times -1} 2x+y=-3 \\ ② 3x-y=4 \rightarrow 3x-y=4 \end{cases}$$

$$\xrightarrow{x=1}$$

$$① 2-y=3$$

$$-y=1$$

$$y=-1$$

$$\boxed{(1, -1)} \checkmark$$

$$16. \begin{cases} ① 5x-4y=-30 \xrightarrow{\times 3} 15x-12y=-90 \\ ② 2x+3y=-12 \xrightarrow{\times 4} 8x+12y=-48 \end{cases}$$

$$\underline{23x=-138}$$

$$x=-6$$

$$② -12+3y=-12$$

$$3y=0$$

$$y=0$$

$$\boxed{(-6, 0)} \checkmark$$

$$17. \begin{cases} ① -x-5y=30 \xrightarrow{\times 2} -2x-10y=60 \\ ② 2x-7y=25 \rightarrow 2x-7y=25 \end{cases}$$

$$\underline{-17y=85}$$

$$y=-5$$

$$② 2x+35=25$$

$$2x=-10$$

$$x=-5$$

$$\boxed{(-5, -5)} \checkmark$$

$$18. \begin{cases} ① -3x+y=-3 \\ ② 2x-5y=-11 \end{cases}$$

$$① y=3x-3$$

$$② 2x-5(3x-3)=-11$$

$$2x-15x+15=-11$$

$$-13x=-26$$

$$x=2$$

$$① -6+y=-3$$

$$y=3$$

$$\boxed{(2, 3)} \checkmark$$